The Effect of On- and Off-Ramps Positions on The Traffic Flow Behaviour

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February 2, 2008

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Abstract

The effect of the position of on-ramp and off-ramp i_1 and i_2, respectively, on the one dimensional-cellular automaton traffic flow behaviour, is investigated numerically. The injection rates at i_1 and i_2 are α_0 and β_0, respectively. However, in the open boundary conditions, with injecting and extracting rates α and β and using parallel dynamics, several phases occur; namely, low density phase (LDP), intermediate density phase (IDP), plateau current phase (PCP) and high density phase (HDP). It is found that phase diagrams exhibit different kind of topologies. For intermediate value of extracting rates β_0 and β and low value of α, (i_1, α_0) phase diagram provides LDP-IDP, LDP-PCP, IDP-PCP, and PCP-HDP transitions, and critical end points. The off-ramp position is located to the middle of the road. By increasing β_0 and β, the IDP desappears. For high value of β, only LDP-HDP persists.

Pacs number : 05.40.-a, 05.50. +q, 64.60.Cn, 82.20.wt
Keywords : Traffic flow, phase diagrams, Cellular automata, on- and off-ramp, parallel update.

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1 Introduction

During the last years, the field of transport have attracted several researchers [ ]. This interest is due primarily to the fact that transportation problems are related to the global behaviour of systems with many elements interacting at short distances, such as the vehicles traveling on the streets, or informations which travel over the internet network. In particular, the investigation of open traffic systems with on- and off-ramps is quite popular at the moment [ ]. One reason for this is the impact of the understanding of varying the different flow ratesin order to optimize the total flow or trip times.

Among the different methods of investigation and simulation of highway traffic, assymetric simple exclusion process (ASEP) is the most promesing [ ]. Indeed, ASEP is the simplest driven diffusive system where particles on a one-dimensional lattice hop with asymetric rates under excluded volume constraints.

The question we want to answer is the following: let us suppose we have a highway running in an urban conglomeration and that there are an access from urban conglomeration to the highway and an exit from the highway. We want to understand where the access and the exit positions must be located in order to maximize the flux of cars in the road. Our aim in this paper is to study the effect of the on-ramp and off-ramp positions on the one dimensional-cellular automaton traffic flow behaviour in the open boundaries case. Depending on the injecting and extracting rate values, an adequate localization of the on- and off-ramp positions leads to the appearance of new phases and topologies. Moreover, to compare our results to those where only one off-ramp was taken into account [30], quantitative differences can be understood from the behaviour of average density, current and phase diagrams for different parameters.

The paper is organised as follows: Model and method are given in section 2; section 3 is reserved to results and discussion; the conclusion is presented in section 4.

2 Model

We consider a one-dimensional lattice of length L. Each lattice site is either empty or occupied by one particle. Hence the state of the system is defined by a set of occupation numbers τ_1,τ_2,...,τ_L, while $\tau_i = 1$ ($\tau_i = 0$) means that the site i is occupied (empty). We suppose that the main road is single lane, an on-ramp and an off-ramp connect the main road only on single lattice i_1 for entry and on single lattice i_2 for way out. During each time interval Δt, each particle jump to the empty adjacent site on its right and does not move otherwise ($i \neq i_2$). Δt is an interesting parameter that enables the possibility to interpolate between the cases of fully parallel ($\Delta t = 1$) and random sequential ($\Delta t \rightarrow 0$) updates [29]. Particles are injected, by a rate $\alpha \Delta t$, in the first site being to the left side of the road if this site is empty, and particles enter in the road by site i_1, with a probability $\alpha_0 \Delta t$ without constraint, if this site is empty. While, the particle being in the last site on the right can leave the road with a rate $\beta \Delta t$ and particles removed on the way out with a rate $\beta_o \Delta t$. At site i_1 (i_2) the occupation (absorption) priority is given to the particle which enter in the road (particle leaving the road). Hence the cars, which are added to the road, avoid any collision.
In our numerical calculations, the rule described above is updated in parallel, $\Delta t = 1$, i.e. during one update step the new particle position do not influence the rest and only the previous positions have to be taken into account. During each of the time steps, each particle moves one site unless the adjacent site on its right is occupied by another particle. The advantage of parallel update, with respect to sublattice or sequential update is that all sites are equivalent, which should be the case in realistic model with translational invariance.
In order to compute the average of any parameter w ($< w >$), the values of $w(t)$ obtained from 5×10^4 to 10^5 time steps are averaged. Starting the simulations from random configurations, the system reaches a stationary state after a sufficiently large number of time steps. In all our simulations, we averaged over $60 - 100$ initial configurations. For the update step, we consider two sub steps as shown in figure 1:
In the first sub step, the sites i_1 and i_2 are updated and in the second half, the chain updates. Thus if the system has the configuration $\tau_1(t)$, $\tau_2(t)$,...,$\tau_L(t)$ at time t it will change at time $t + \Delta t$ to the following:
For $i = i_1$,

$$\tau_i(t + \Delta t/2) = 1 \tag{1}$$

with probability

$$q_i = \tau_i(t) + [\alpha_0(1 - \tau_i(t)) - \tau_i(t)(1 - \tau_{i+1}(t))]\Delta t \tag{2}$$

and

$$\tau_i(t + \Delta t/2) = 0 \tag{3}$$

with probability $1 - q_i$. Where i_1 and α_0 denote the position of the entry site and the injection rate, respectively.
For $i = i_2$,

$$\tau_i(t + \Delta t/2) = 1 \tag{4}$$

with probability

$$q_i = \tau_i(t) + [\tau_{i-1}(1 - \tau_i) - \beta_0 \tau_i(t)]\Delta t \tag{5}$$

and

$$\tau_i(t + \Delta t/2) = 0 \tag{6}$$

with probability $1 - q_i$. Where i_2 and β_0 denote the position of the absorbing site and the absorbing rate, respectively.
For 1<i<L with $i \neq i_1$ and $i \neq i_2$,

$$\tau_i(t + \Delta t) = 1 \tag{7}$$

with probability

$$q_i = \tau_i(t) + [\tau_{i-1}(t)(1 - \tau_i(t)) - \tau_i(t)(1 - \tau_{i+1}(t))]\Delta t \tag{8}$$

and

$$\tau_i(t + \Delta t) = 0 \tag{9}$$

with probability $1 - q_i$.

For $i = 1$,

$$\tau_1(t + \Delta t) = 1 \tag{10}$$

with probability

$$q_1 = \tau_1(t) + [\alpha(1 - \tau_1(t)) - \tau_1(t)(1 - \tau_2(t))]\Delta t \tag{11}$$

and

$$\tau_1(t + \Delta t) = 0 \tag{12}$$

with probability $1 - q_1$.
For $i = L$,

$$\tau_L(t + \Delta t) = 1 \tag{13}$$

with probability

$$q_L = \tau_L(t) + [\tau_{L-1}(t)(1 - \tau_L(t)) - \beta \tau_L(t)]\Delta t, \tag{14}$$

and

$$\tau_L(t + \Delta t) = 0 \tag{15}$$

with probability $1 - q_L$.

3 Results and Discussion

As we have mentioned previously, our aim in this paper is to study the effect of the positions of the on- and off-ramps i_1 and i_2, respectively, for different values of α, β_0 and β, on the average density and flux in chain. The study is made in the open boundary conditions case. α_0 and α denote the injecting rates at first site (i = 1) and at site i_1, respectively. β_0 and β are the extracting rates at site i_2 and at the last one (i = L), respectively. The length of the road studied here is L=1000.

The figures 2(a) and 2(b) give respectively the variation of the average density ρ and average current J versus the injecting rate α_0 for several values of the on-ramp position i_1. These figures are given for $\alpha = 0.1$, $\beta = 0.1$ and $\beta_0 = 0.4$. However, when the on-ramp is located upstream of the off-ramp, the system studied exhibits four phases, depending of the behaviours of the density, ρ, and the current, J. Namely: i) The low density phase (LDP), where the averages density and current increase when increasing the rate of injected particles α_0. ii) The intermediate density phase (IDP) characterised by a smoothly increase of the density and average current. iii) The plateau current phase (PCP) for which the density and current are constant in a special interval of α_0. iv) The high density phase (HDP) in which, for high values of α_0, the current decreases and the density reaches its maximum value and remains constant. On the other hand, when i_1 is located downstream from i_2, the IDP desappears. In addition, when increasing i_1 for a given value of α_0, the figure 2(a) shows that the average density is constant in LDP, increases in IDP and PCP then decreases in HDP. The figure 2(a) exhibits an inversion point situated at the PCP-HDP transition. Moreover, the figure 2(b) shows that the average current deceases by increasing i_1, for any value of α_0.

For $i_1 < i_2$, we note that the IDP, which doesn't appear in the model where only the off-ramp is taken into account [30], occurs for the intermediate values of α_0 ($\alpha_{0c1} < \alpha_0 < \alpha_{0c2}$). α_{0c1} and α_{0c2} correspond to the transition between LDP-IDP and IDP-PCP, respectively. While the PCP arises between two critical values α_{0c2} and α_{0c3} of injecting rate α_0. Where α_{0c3} corresponds to the PCP-HDP transition. Note that the transitions which occur at α_{0c3} disappears when i_1 is located after i_2 (Figure 2a). Now, in order to have a suitable criterion for determination of the nature of the transition, we identify the first order transition (abrupt transition) by the jump in the average density or by the existence of a peak in the derivative of $\rho(\alpha_0)$ with respect to α_0. The jump in density corresponds to a first order transition [29]. This means that the above transitions are of first order type.

Collecting the results illustrated in figures 2(a) and 2(b), the four regions are given on the phase diagram (i_1,α_0) shown in Figure 2(c). Beside this, such phase diagram exhibits four critial end points, around which there is no distnction between the phases. This critical end points are indicated by CEP.

For low values of α, β and β_0 ($\alpha = 0.1$, $\beta = 0.1$ and $\beta_0 = 0.1$), The (i_1, α_0) phase diagram is presented in figure 3. This figure exhibits only tow first order phase transitions. Namely, LDP-PCP, PCP-HDP transitions. The later one can be founed by variying α_0,for a given value of i_1 lower than i_2, or at $i_1 = i_2$, for $0.45 < \alpha_0 < 0.95$. Moreover, the figure 3 exhibits two critical end points. The comparison of figures 2 and 3 highlight the effect of β_0 on the (i_1, α_0) phase diagram. Indeed, for intermediate value of β_0, the IDP arises.
For a sufficiently large value of β_0, the critical end points desapears, as shown in figure 4. This figure is given for $\alpha = 0.1$, $\beta = 0.1$ and $\beta_0 = 0.8$.
The figure 5 give the (i_1, α_0) phase diagram for $\alpha = 0.1$, $\beta_0 = 0.1$ and $\beta = 0.3$. In this case, the system exhibits four phases and three critical end points. From figures 2c and 5, we deduce that the IDP arises for intemediate values of extracting rates β or β_0, when i_1 is upstream of $_2$.

4 Conclusion

Using numerical simulations, we have studied the effect of the on- and off-ramp positions on the traffic flow behaviour of a one dimensional-cellular automaton, with parallel update. Depending on the values of α, β and β_0, the (i_1, α_0) phase diagram exhibits different topologies. The IDP occurs only at special positions of on- and off-ramps with special values of extracting rates β and β_0 and injecting rates α and α_0. The transition between different phases are of first order. Furthermore, the system exhibits critical end points in the (i_1, α_0) plane in the case of moderate values of β and β_0 and small value of α.

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Figure captions:

Fig.1: Example of configuration obtained after two sub steps for system size $L = 15$.
Fig.2: For $\alpha = 0.1$ and $\beta = 0.1$; (a) Average density ρ versus the injection rate α_0; (b) Variation of the average current as a function of α_0; (c) Phase diagram (β_0,α_0). The number accompanying each curve, in (a) and (b), denotes the values of β_0.
Fig.3: Phase diagram (β,α_0) for $\alpha = 0.1$ and $\beta_0 = 0.1$.
Fig.4: Phase diagrams (β_0,α_0) for $\beta = 0.1$; (a) $\alpha = 0.2$; (b) $\alpha = 0.4$; (c) $\alpha = 0.5$.